Phase transition in d-dimensional long-range interacting systems

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We present new iterative method for the derivation of a one-point distribution function in the equilibrium state. For derivation of the distribution function, we must solve the Lane-Emden equation. In the general case, we solve the equation with an iterative method. However, the traditional method does not ensure convergence of the algorithm, we cannot often obtain solutions. In order to obtain a stable stationary distribution function, we have proposed a new iterative method. Our method ensures entropy increase and convergence of the algorithm. Furthermore, our method can obtain the distribution function quickly [1].

Here we present the phase transition in long-range interacting systems. The HMF model describes the motion of globally coupled particles on a 1D circle. Nevertheless, the interaction is described with only one cosine function; both the dynamical and the thermodynamical properties of this mode are quite various and complicated. For the HMF model, an extension for the 2D model had been proposed [2]. We have extended the HMF model for 3D and 4D models. The Hamiltonian of the HMF models is written as

$$H = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2} + V\,, \quad (1)$$

$$V = \frac{1}{2N}\left[2^d - \prod_{a=1}^{d}\sum_{i,j}^{N}\left(1+\cos x_{ij}^{(a)}\right)\right]\,, \quad (2)$$

where N is the number of particles. d is the spacial dimension.

We take the thermodynamical limit ($N \to \infty$) and analyse the equilibrium state in both microcanonical and canonical ensembles. The results for the microcanonical and canonical ensembles are shown in Tables 1 and 2, respectively.

Table 1
The relation between spacial dim., order of phase transition, critical energy, and existence of negative specific heat in a microcanonical ensemble.

spacial dim.	phase transition	U_c	negative specific heat
1	2nd	0.75	No
2	2nd	2.00	Yes
3	1st	4.28	Yes
4	1st	8.66	Yes

Table 2
The relation between spacial dim., critical energy, critical temperature, and change of concavity in a canonical ensemble.

spacial dim.	U_{low}	U_{high}	T_c	U_{top}
2	1.6160	2.0413	0.54137	1.795
3	2.7430	4.5620	0.70811	3.517
4	4.852	9.5409	1.0206	6.836

The authors thank the Supercomputer Center, Institute for Solid State Physics, University of Tokyo for the use of the facilities.

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