Recent Progress in Astrophysical MHD

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Abstract

Studying the multidimensional, time-dependent and/or highly nonlinear dynamics of astrophysical plasmas usually requires numerical methods, however developing accurate and robust algorithms for compressible MHD and/or radiation MHD is still an active area of research. Recent advances in the development of numerical algorithms are described, and results from applications of the methods to the study of a variety of problems in astrophysics is discussed.

Key words: Numerical methods, MHD

1. Introduction

Numerical methods are essential for the study of nonlinear and time-dependent magnetohydrodynamics (MHD) in astrophysics. It is important to continue the development of more accurate and robust numerical algorithms in order to enable investigation of new and more complex phenomena. This short paper summarizes some of our developments, implemented in a new code for astrophysical MHD called Athena. Much more detail can be found in [1] and [2] (hereafter GS05 and GS07 respectively)

2. Method

Athena solves the equations of ideal MHD

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \mathbf{v}) = 0 \qquad (1)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla\cdot(\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B}) + \nabla P^* = 0 \qquad (2)$$

$$\frac{\partial E}{\partial t} + \nabla\cdot((E + P^*)\mathbf{v} - \mathbf{B}(\mathbf{B}\cdot\mathbf{v})) = 0 \qquad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times(\mathbf{v}\times\mathbf{B}) = 0 \qquad (4)$$

where P^* is the total pressure (gas plus magnetic), E is the total energy density, and we have chosen a system of units in which the magnetic permeability $\mu = 1$. The other symbols have their usual meaning. We use an ideal gas equation of state for which $P = (\gamma - 1)\epsilon$, where γ is the ratio of specific heats, and the internal energy density ϵ is related to the total energy E via

$$E \equiv \epsilon + \rho(\mathbf{v}\cdot\mathbf{v})/2 + (\mathbf{B}\cdot\mathbf{B})/2\ . \qquad (5)$$

Athena is based on higher-order Godunov techniques. The above equations are discretized using a control-volume approach, with volume averages of the density, total energy, and momentum stored at cell-centers, and area averages of the magnetic field stored at cell-faces. The constrained transport (CT) method [3] is used to ensure the face-centered magnetic field remains divergence-free. The critical components of the algorithms are summarized in the following subsections.

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2.1. *Spatial Reconstruction*

To compute the values of the conserved quantities at the left- and right-sides of each cell interface, we use the piecewise parabolic method (PPM) of [4]. However, we have shown that for MHD, it is necessary to include transverse gradients of the magnetic field in the PPM reconstruction algorithm, that is in the calculation of the interface states of B_x it is necessary to include terms proportional to $\partial B_y/\partial y$ and $\partial B_z/\partial z$. The proof that these terms are required, and their derivation, are given in GS05 (for 2D) and GS07 (for 3D).

We use characteristic interpolation in the primitive variables, which requires an eigenvalue decomposition of the MHD equations, but produces less oscillations near strong shocks.

2.2. *Riemann Solvers*

To compute the fluxes of conserved quantities from the left- and right-states constructed above, a variety of Riemann solvers can be used. We have implemented Roe's linearized solver [5], and various forms of the HLL solver [6], as well as exact solvers for hydrodynamics. Roe's solver is accurate for all wave families, but often fails for strong rarefactions. Versions of the HLL solvers that do not include the contact wave are too diffusive for practical applications. We find the extension of HLL that includes the contact wave (called HLLC in [6] for hydrodynamics, and HLLD in [7] for MHD) is the most efficient, robust, and accurate.

2.3. *Constrained Transport*

By discretizing the magnetic field using face-centered area-averages, we can adopt the CT difference formulae to ensure the divergence-free constraint is enforced. However, as shown in GS05, the coupling of CT with a Godunov method requires care in the construction of the edge-centered electric fields from the face-centered fluxes returned by the Riemann solver (see above). For example, simple arithmetic averaging of fluxes to cell edges results in a numerical method which is unstable (see GS05). Fortunately, by treating the projection of the electric field to corners as a reconstruction step, it is straightforward to derive simple formulae for the edge-centered electric field, again see GS05 and GS07 for details.

2.4. *Directionally unsplit integrators*

In GS05 we showed that directionally-split integration methods, which are extremely powerful for hydrodynamics, cannot be used in MHD without violating the divergence-free constraint. For this reason, we adopt the unsplit corner transport upwind (CTU) method of [8] to integrate the cell-centered quantities.

Even so, substantial modifications to CTU are required to make it work with MHD using CT. In particular, the transverse flux gradients used to correct the left- and right-states with in multidimensions must be modified with "MHD source terms", that is terms proportional to $\partial B_x/\partial x$, $\partial B_y/\partial y$ and $\partial B_z/\partial z$. The exact form of the necessary terms are given in GS05 and GS07.

3. Tests

Figure 1 plots the L1 error norm for linear amplitude fast, slow, Alfven, and entropy waves in a three-dimensional domain using a resolution of $2N \times N \times N$ grid points, where $N = 8, 16, 32, 64$, and 128. The wavevector is inclined at angles of α and β in the $x-y$ and $x-z$ planes respectively, where $\sin\alpha = 2/3$ and $\sin\beta = 2/\sqrt{5}$. This ensures the test is truly multidimensional. The HLLD Riemann solver is used. This demonstrates Athena converges at second-order for all wave families in 3D. The waves propagate in a uniform medium for one crossing time of the domain. The details of the test are given in [2].

A variety of other test problems have been used to develop the algorithm. Some of the most challenging include

(i) *Advection of a Field Loop.* This is a challenging test for Godunov schemes using CT, since the field will be distorted or the method will be unstable if the edge-centered electric fields are not consistent.

(ii) *Nonlinear Alfven Wave.* This tests convergence rate for nonlinear amplitude, smooth solutions. It also tests stability of the algorithm.

(iii) *Shocktubes rotated to grid.* This tests that the multidimensional algorithm can correctly propagate plane waves at any angle.

(iv) *MHD Blast wave.* This tests whether the method is robust, and whether it can hold spherical symmetry.

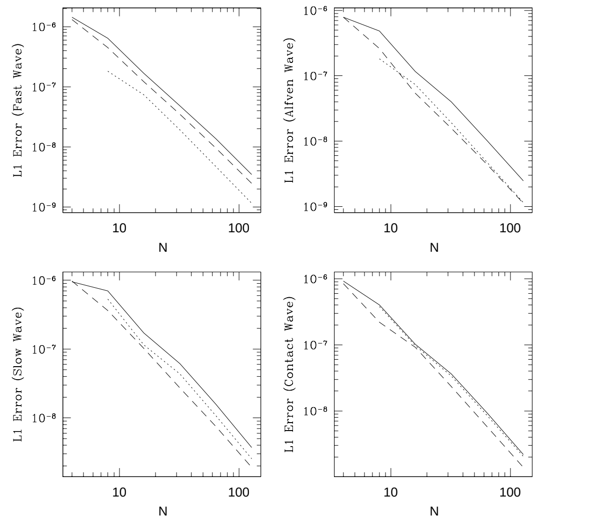

Fig. 1. L1 error norm for each wave family in MHD as a function of grid resolution N. The solid line uses second-order (piecewise linear) reconstruction, the dashed line uses PPM for spatial reconstruction, while the dotted line is the equivalent test in one-dimension using second-order reconstruction.

A complete description of all the tests used can be found in GS05, GS07, and [9]

4. First Applications

Athena has been developed to study the dynamics of supersonic turbulence in the interstellar medium, and the dynamics of magnetized accretion flows. However, a wide variety of other applications is possible. For example, Figure 2 compares the structure of the Rayleigh-Taylor (RT) instability in the nonlinear regime in hydrodynamics and MHD.

The initial conditions are two inviscid, perfectly conducting fluids of constant density separated by a contact discontinuity perpendicular to the effective gravity g, with a uniform magnetic field $\mathbf{B}$ parallel to the interface. RT modes parallel to the field with wavelengths smaller than $\lambda_c = \mathbf{B}\cdot\mathbf{B}/(\rho_h - \rho_l)g$ are suppressed (where ρ_h and ρ_l are the densities of the heavy and light fluids respectively), whereas modes perpendicular to $\mathbf{B}$ are unaffected. In [10], we have studied the evolution of strong fields with λ_c varying between 0.01 and 0.36 of the horizontal extent of the computational domain. Even a weak field produces tension forces on small scales that are significant enough to reduce shear (as measured by the distribution of the amplitude of vorticity), which in turn reduces the mixing between fluids, and increases the rate at which bubbles and finger are displaced from the interface compared to the purely hydrodynamic case. This is clearly evident in Figure 2. For strong fields, the highly anisotropic nature of unstable modes produces ropes and filaments. However, at late time flow along field lines produces large scale bubbles. These simulations are relevant to Z-pinch experiments, and a variety of astrophysical systems.

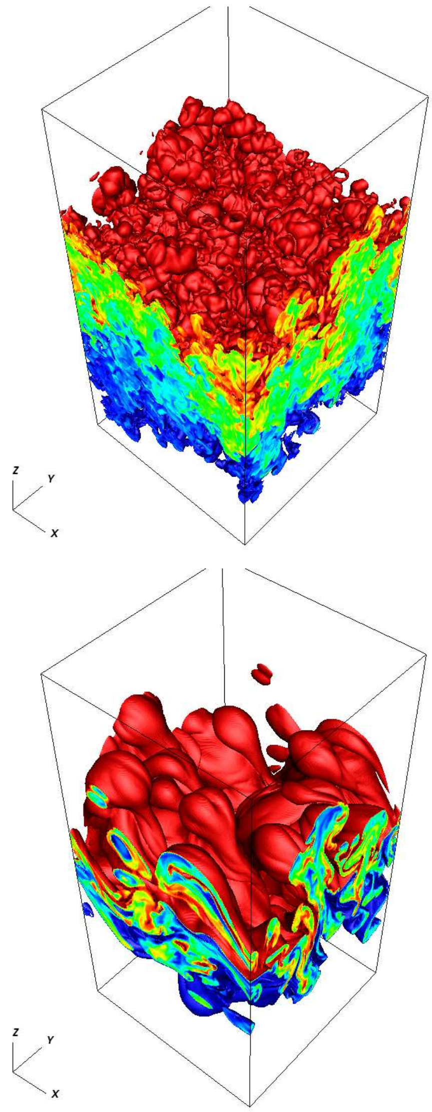

Fig. 2. Isosurfaces of the density in the hydrodynamic *(top)* and MHD *(bottom)* evolution of the RT instability.

5. Summary

A variety of other algorithmic developments, not described here, have been made. These include nested and adaptive mesh refinement for MHD based on CT and CTU, and self-gravity. Further details of these topics will be published in the future. In fact, the primary motivation for development of Athena was the need to adopt single-step Eulerian methods for AMR.

Our primary focus at the moment is using the unigrid, and fixed nested grid, version of Athena for astrophysical applications. Current applications include (1) the dynamics of vortices in hydrodynamic disks [11], (2) the hydrodynamics of colliding stellar winds [12], (3) shock interaction with magnetized clouds in three-dimensions [13], (4) the nonlinear evolution and saturation of the magneto-thermal instability [14], (5) the magnetic Rayleigh-Taylor instability [10], and (6) evolution of ionization fronts in a magnetized, turbulent medium [15]. Many further applications are planned.

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