The magnetized plasma-wall transition (PWT) and its relation to fluid boundary conditions

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Abstract

The magnetized plasma-wall transition (PWT) region typically exhibits three characteristic subregions: the "Debye sheath", the "magnetic presheath", and the "collisional presheath". The fluid boundary conditions for transport codes (simulating, e.g., the scrape-off layer (SOL) of a tokamak) are usually applied at the "magnetic presheath entrance", where in the simplest model the ion velocity parallel to the magnetic field equals the local sound velocity. After reviewing the basic time-independent and collisionless models of the magnetized PWT, various extensions will be discussed which are due to $\mathbf{E} \times \mathbf{B}$, ∇B and diamagnetic drifts, non-uniformity of the electric field parallel to the wall, and turbulence effects. In practically all cases considered, quantitative results can be obtained only by massive application of numerical methods of solution.

Key words: Plasma sheath; plasma-wall transition

1. PWT fundamentals and overview

The steady state of a plasma device is largely determined by the transition zone between the plasma and the bounding material walls (limiters, divertors, outer walls, etc.). In the presence of a magnetic field $\mathbf{B}$ inclined obliquely to an absorbing solid surface, a typical plasma-wall transition (PWT) shows a potential that decreases monotonically towards the surface, thus accelerating the ions towards the latter, and exhibits three distinct subregions: (i) the Debye sheath (DS), which is adjacent to the wall and whose thickness is of the order of the electron Debye length, λ_{De}; (ii) the magnetic presheath (MP), which is located upstream of the DS and whose thickness is of the order of the ion gyroradius, ρ_i; and (iii) the collisional presheath (CP), which is located upstream of the MP and whose thickness is less (or much less) than the dominant (i.e., the smallest relevant) collisional length λ_c. If the ordering $\lambda_{De} \ll \rho_i \ll \lambda_c$ ("asymptotic three-scale limit") is satisfied, the DS and the MP are collisionless, the MP and the CP are quasineutral, and the MP is well separated from the DS and the CP. If it is not, the three PWT subregions (DS, MP and CP) are not clearly distinguished and separated, as is explicitly stated at the end of Sec.3.

In the DS, the ion space charge density exceeds the electron one. In the asymptotic three-scale limit, monotonicity of the potential requires that at the upstream end of the DS (DS entrance, DSE) the ion velocity perpendicular to the solid surface must be equal to (or, in exceptional cases, larger than) the ion-sound velocity c_s (Bohm criterion) [1–3].

In the MP, the electric field is strong enough to deflect the ion orbits in such a way that the ion velocity component perpendicular to the wall can ful-

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fill the Bohm criterion at the DSE. The physical conditions at the upstream end of the MP (MP entrance, MPE) are usually taken as boundary conditions for fluid codes simulating magnetized bounded plasmas (e.g., the tokamak scrape-off layer, SOL) [6]). In the simplest MP model, which was given by Chodura [4,5], neither drifts nor gradients perpendicular to the wall are considered. Assuming the MP to be quasineutral and collisionless (which amounts to the asymptotic three-scale limit), Chodura found that the MPE lies at infinity (meaning, in practice, at a distance well greater than ρ_i) and is characterized by the "Bohm-Chodura criterion" $v_{\shortparallel 0} \geq c_s$, where $v_{\shortparallel 0}$ is the ion fluid velocity parallel to $\mathbf{B}$ at the MPE. Customarily, the marginal form of this criterion, $v_{\shortparallel 0} = c_s$, is invoked to define the MPE. This is even done for situations in which the asymptotic three-scale limit is not well satisfied. A more consistent definition of the MPE for such cases would certainly be desirable.

In more realistic and complex models, the parallel velocity at the MPE, $v_{\shortparallel 0}$, is (in the three-scale limit) not simply given by c_s but contains additional terms, cf. Secs. 2, 3 and 4.

2. Drift effects

In the simplest picture of the tokamak SOL, the plasma particles are removed exclusively by transport *along* the magnetic field lines to the solid surfaces of limiter or divertor plates. In a more refined description, however, one also has to consider particle losses *across* the magnetic field due to first-order particle drifts, i.e., the $\mathbf{B} \times \nabla p_i$ (diamagnetic), $\mathbf{B} \times \nabla B$, and $\mathbf{E}_c \times \mathbf{B}$ drifts, where p_i is the ion pressure and $\mathbf{E}_c$ is the "cross" electric field (i.e., the electric-field component perpendicular to $\mathbf{B}$) [6,7].

The diamagnetic drift is almost entirely divergence-free, so that in the DS region, which is characterized by strong pressure gradients normal to the wall, the poloidal component of the diamagnetic particle flux entering at the MPE is diverted into the direction along the target surface. This effect creates boundary drift flows along the surface and prevents the particles from hitting the latter, so that, although the diamagnetic drift velocity enters the modified Bohm-Chodura condition at the MPE (see (1) below), the diamagnetic fluxes must not be included in the total fluxes hitting the wall.

The $\mathbf{B} \times \nabla B$ drift is negligibly small because it scales as $1/R$, with R the major radius of the tokamak.

The changes to the Bohm-Chodura criterion due to the poloidal component of the $\mathbf{E}_c \times \mathbf{B}$ drift at the MPE can be calculated. This drift can cause the parallel fluid velocity $v_{\shortparallel 0}$ to be different from c_s.

In summary, a weak electric field parallel to $\mathbf{B}$ still present in the CP accelerates the ions up to a certain velocity $v_{\shortparallel 0}$ which is necessary for the existence of a quasineutral collisionless MP and defines the MPE. Taking into account the drift effects addressed here, one finds for this velocity the equation

$$v_{\shortparallel 0} \sin\theta - \frac{E_y \cos\theta}{B} + \frac{\cos\theta}{enB}\frac{\partial p_i}{\partial y} = c_s \sin\theta, \qquad (1)$$

where the y direction is perpendicular to both $\mathbf{B}$ and the normal to the wall surface, and $\sin\theta = B_\theta / B$ with B_θ is the poloidal magnetic field. [6,7]. This equation, in which the drift effects are represented by the second and third terms on the left-hand side, may be considered as a modified Bohm-Chodura criterion. Note that in the absence of drifts we recover the classical Bohm-Chodura criterion $v_{\shortparallel 0} = c_s$, and that due to the presence of the drift terms $v_{\shortparallel 0}$ may be greater or less than c_s.

3. Effect of nonuniform cross electric field

Here we also consider the effect of nonuniformity of the cross electric field $\mathbf{E}_c$ [8]. The assumption of uniform $\mathbf{E}_c$ used in the previous discussion is not quite correct because the electric-field component parallel to the wall vanishes at the conducting wall and hence, has a strong normal gradient in the direction normal to the wall. Neglecting the ion collisions with neutrals and the diamagnetic drift we find that the ion velocity at the MPE equals

$$v_{\shortparallel 0} = c_s \left(\sqrt{1+\eta^2} - \eta\right) + \frac{E_y}{B}\cot\theta, \qquad (2)$$

with

$$\eta = \pm\frac{1}{2}\frac{\cot\theta}{1+T_i/T_e}\frac{\rho_i}{L_y}\sqrt{1+T_e/T_i},$$

$$L_y^{-1} = \left|\frac{\partial_z E_y}{E_z}\right|_{z\to-\infty}.$$

This condition reduces to the Bohm-Chodura condition (1) only in the limit $\eta \ll 1$. By means of the relation $\partial E_y/\partial x = \partial E_x/\partial y$ one can find by means of simple estimates that $c_s\eta$ (which is proportional to the gradient of the $\mathbf{E}_c \times \mathbf{B}$ drift) has the same order as the last term in (2) (which equals the $\mathbf{E}_c \times \mathbf{B}$

drift), so that in general both terms have to be taken into account simultaneously.

In order to verify the analytic results outlined here we have made PIC simulations using the 1d3v code BIT1 [8], developed at Innsbruck University on the basis of the XPDP1 code from U.C. Berkeley [9]. We consider a hydrogen plasma. In the center of the simulation domain we implemented an ambipolar particle (ion and electron) source. At each side of the system we have an absorbing wall. Our model includes the DS and the MP as well as the CP. For the charged-neutral particle collisions in the CP we consider elastic and charge-exchange collisions between hydrogen ions and hydrogen atoms, with constant cross sections (10^{-18}m^2 and 0.4×10^{-18}m^2 respectively). The plasma parameters are chosen so as to be relevant to tokamak plasmas: ($B = 1$T, $\theta = 5°$), in the source region the plasma density and particle temperatures are about 10^{18}m^{-3} and 30 eV, respectively. The atomic hydrogen density is 10^{19}m^{-3}. The simulated system is about $L = 45\rho_i$ long and contains 800 spatial grid cells. In order to ensure high accuracy, a large number of simulation particles is used, on average about 350 per spatial grid cell. During the simulation Maxwell-distributed electrons and ions are injected into the source region. After a few ion transit times $L/\sin\theta\sqrt{T_i/m_i}$, the system reaches a stationary state.

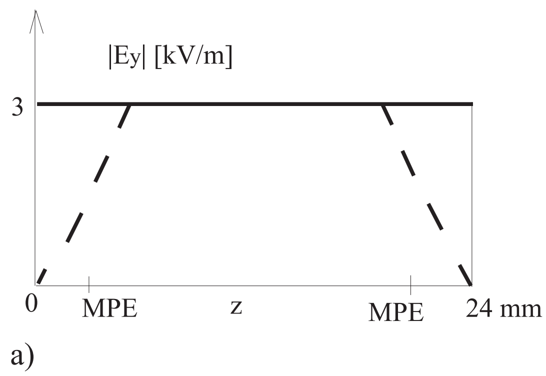

Fig. 1. Profile of the applied E_y: $E_y = const$ (solid line), E_y with gradient (dashed line).

We have made two simulations with imposed electric fields parallel to the wall (E_y): (a) with constant E_y ($E_y \sim c_s B\sin\theta/c_s$), and (b) with E_y constant inside the CP and linearly decreasing in the MP (Fig. 1). In case (b) the E_y gradient starts about one ion gyroradius in front of the MPE. The $\mathbf{E}_c\times\mathbf{B}$ drift causes a asymmetry of all plasma parameters, as at the right-hand side wall (where the $\mathbf{E}_c\times\mathbf{B}$ drift is directed towards the center) the plasma is practically detached and the related numerical fluctuations (due to the low number of simulation particles) does not allow quantitative analysis. Profiles of the ion fluid velocity $v_{\shortparallel 0}$ at the left-hand side MPE (where the $\mathbf{E}_c\times\mathbf{B}$ drift is directed towards the wall) for different E_y are given in Fig. 2.

For the constant E_y ($\eta = 0$), the ion fluid velocity at the MPE, $v_{\shortparallel 0}$, is larger by a factor of 1.5 than for the nonuniform E_y ($\eta \neq 0$). These values are in good (qualitative) agreement with analytic estimates obtained from the boundary condition (2).Hence, the analytic and PIC simulation results demonstrate that the gradient of the $\mathbf{E}_c\times\mathbf{B}$ drift at the MPE cannot be neglected and the correct boundary conditions for the ion fluid velocity should include the terms appearing in (2).

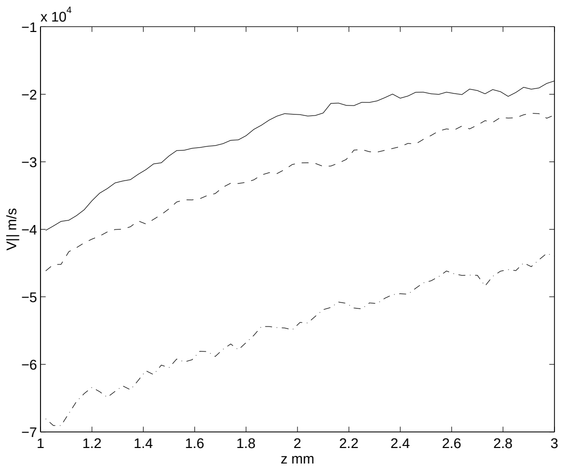

Fig. 2. Profiles of the parallel component of the ion velocity ($v_{\shortparallel} = 0$) near the MPE. The MPE is located at 2 mm. Solid, dashed, and dotted lines correspond to nonuniform E_y (the gradient of E_y starts at 2.4 mm), to constant $E_y \neq 0$, and to $E_y = 0$, respectively.

4. Turbulence effects

In [10], a novel fluid model of the MP in a turbulent boundary plasma has been developed, which self-consistently takes into consideration turbulent-transport corrections of the classical fluid transport equations customarily used for modelling boundary plasmas. The main scientific motivations for this study were the failure of the previous theoretical models to successfully explain many experimental results and a need for improved, more realistic fluid boundary conditions near solid material walls in contact with the plasma. These boundary conditions are particularly important in the fluid transport models of bounded plasma systems and related computer codes, especially in the fluid transport codes for tokamak boundary plasma modelling and future fluid transport codes for integrated tokamak modelling.

The turbulent particle and momentum sources in the fluid transport equations have been derived from basic principles, i.e., by means of the ensemble averaging procedure from the statistical theory of plasma turbulence. This approach can also be used for deriving turbulent transport corrections to the fluid transport equations used for modelling the CPs or bulk plasmas in bounded plasma systems, where particle collisions and turbulent fluctuations are both important for the macroscopic transport. Unfortunately, this procedure usually yields some complicated expressions for the correlations between the fluctuating quantities, which cannot easily be expressed as functions of the average values of the plasma parameters. Possible practical solutions for this problem are (i) to replace these unknown correlations with some phenomenological expressions based on analogy with classical transport, as was done in [10], (ii) to use experimentally determined expressions for these correlations, which, in principle, should be intrinsically generic (i.e., not restricted to only some special cases), or (iii) to use semi-empirical expressions, which are combinations of theoretical results and experimental data, and should also be generic.

The main results of [10], which are based on the "asymptotic three-scale limit" addressed in Sec. 1 above, can be summarized as follows: (a) The total ion flux density perpendicular to the wall, which is the sum of the regular and the turbulent ion flux densities perpendicular to the wall, is conserved in the planar MP. (b) The fluid formulation of the Bohm criterion at the DSE of a turbulent plasma in a magnetic field has been derived, cf. Eq. (32) of [10]. (c) The generalized Bohm-Chodura criterion at the MPE of such a plasma, which is the most important result in our present context, has been derived in the form $\Gamma_{i,z} \geq n^r c_s \sin\vartheta + \Gamma_{i,z}^{turb}$, with $\mathbf{\Gamma}_i^{turb} = -\overleftrightarrow{D} \cdot \nabla n^r + \boldsymbol{v}^{turb} n^r$. Here, $\mathbf{\Gamma}_i$ and Γ_i^{turb} are the total and turbulent ion flux densities, n^r is the regular (i.e., non-turbulent) plasma density, the subscript z denotes components perpendicular to the wall, and all quantities are evaluated at the MPE. (d) Typical values and profiles of plasma parameters at the entrance of the DS and throughout the MP have been calculated numerically over large ranges of experimentally relevant model parameters (Figs. 2–12 of [10]), thus providing amongst other things a quantitative basis for fluid boundary conditions at the MPE.

If the gradient of n^r is non-zero, and/or if an additional convective turbulent ion flux density $v_z^{turb} n^r$ is present at the MPE, then $\Gamma_{i,z}^{turb}$ is non-zero and can even dominate over the regular transport term in some cases, e.g., for very small grazing angles ϑ.

5. Conclusions and perspectives

In this paper we have reviewed the effects of drifts (Secs. 2 and 3) and of turbulence (4) on the fluid boundary conditions at the MPE, which are the ones relevant for fluid codes simulating, e.g., the tokamak SOL.

The classical particle drifts across the magnetic field can play an essential role in the transport phenomena in the tokamak SOL, especially in the anomalous transport in the divertor region. Therefore the correct formulation of the boundary conditions at the MPE is of crucial importance. In this review we have focused on modifications of the conventional Bohm-Chodura criterion at the MPE by the classical (diamagnetic, $\mathbf{E} \times \mathbf{B}$ and ∇B) drifts and also by the nonuniformity of the $\mathbf{E} \times \mathbf{B}$ drift. It has been shown that the dominant effect near the divertor plates comes from the $\mathbf{E} \times \mathbf{B}$ drift, whereas the influence of ∇B drift is weak due to its smallness and the diamagnetic drift does not contribute to the transport phenomena because it is divergence-free. The analytic and PIC simulation results obtained demonstrate that the gradient of the cross electric field at the MPE cannot be neglected and the correct Bohm-Chodura criterion for the ion fluid velocity is given by Eq. (2).

Regarding turbulence effects, important new results are expressions for the fluid Bohm criterion at the DSE and for the ion flux density perpendicular to the wall throughout the MP. These new results, which exhibit corrections due to the turbulent charged particle transport, can qualitatively explain the fact that, whenever the angle between the magnetic field and the wall is very small (i.e., several degrees) or zero, electric currents, measured by Langmuir probes in the boundary regions of nuclear fusion devices and in various low-temperature plasmas, are anomalously enhanced in comparison with those expected or predicted by other theoretical models.

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