The propagation of a microwave in an atmospheric pressure plasma layer: one and two dimensional numerical solutions

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Abstract

The propagation of a microwave in an atmospheric pressure plasma (APP) layer is described numerically with an integral-differential wave equation in one dimension (normal incident) case and with the Finite Difference Time Domain (FDTD) method in two dimension (oblique incident) case. When the microwave passes through the APP layer, its amplitude and phase of the wave electric field are obviously modulated by both the electron density and the collisions between the electrons and neutrals. The dependencies of the passed wave behaviors (i.e. the phase shift, the reflectivity, the transmissivity and absorptivity) on the APP layer characteristics (width, electron density, and collision frequency) and microwave characteristics (incident angle and polarization) are presented. The Appleton's Equation can be derived from the Wentzel-Kramers-Brillouin (WKB) solution of the integral-differential wave equation and is compared with the one dimensional numerical solution.

Key words: Attenuation of Microwave; Atmospheric Pressure Plasmas

1. Introduction

This work is motivated by a recent interest in using atmospheric pressure plasma (APP) as absorbers or reflectors of microwave energy, depending on the particular application[1–3]. An APP is a kind of cold collisional plasmas, and its properties are very different from those of plasmas generated by low-pressure gas discharges. In the APP, the collision frequency ν_{e0} may be equal or even larger than the incitant microwave frequency f_0. When an EM wave propagates in the APP, collisions between charged particles and neutrals of the APP can obviously attenuate the amplitude and shift the phase in a few wave-periods or wavelengths[4–6]. So, the electron fluid motion equation, which includes the collision friction force, should directly couple to the Maxwell (or wave) equations and could not solve it through the Fourier transformation as usual.

2. One dimension case

When a plane electromagnetic wave is incident normally into (in x direction) an inhomogeneous APP layer, the one dimension model can be used to describe wave propagation behaviors[4]:

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y(x,t)}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial J_y(x,t)}{\partial t} = 0, \quad (1)$$

$$J_y(x,t) = -en(x)u_y(x,t), \quad (2)$$

$$\frac{\partial u_y(x,t)}{\partial t} = -\frac{e}{m_e}E_y(x,t) - \nu_{e0}u_y(x,t). \quad (3)$$

When $\nu_{e0} \ll f_0$, the Fourier transformation (or WKB approximation) can be used to solve Eq.(3):

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$$[E_y, u_y, J_y](x,t) = [E,u,J]exp\left\{i\omega t - i\int_0^x k_r(s)ds\right\}, \quad (4)$$

then

$$J = -i\frac{n_e e^2}{m_e}\frac{1}{\omega - i\nu_{e0}}E, \quad (5)$$

and finally

$$\omega^2 = c^2\left[k_r(x) + ik_i(x)\right]^2 + \frac{\omega\omega_{pe}^2(x)}{\omega - i\nu_{e0}}. \quad (6)$$

Where the $k_r(x)$ and $k_i(x)$ are the real and image part of the wave vector [4,6]. The attenuation and phase shift of the microwave passed through the plasma layer (width d) are

$$\Delta A = \int_0^d [k_{i0} - k_i(x)]dx, \quad (7)$$

$$\Delta\varphi = \int_0^d [k_{r0} - k_r(x)]dx. \quad (8)$$

These are the so called Appleton formula [4,6].

But there is $\nu_{e0} \geq f_0$ in APP, we have to deal with the whole coupled equations (1)-(3) in (x,t) domain. By integrating the motion equation directly, an integral-differential wave equation can be derived from Eqs.(1)-(3)

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y(x,t)}{\partial t^2} - \frac{\omega_{pe}^2(x)}{c^2}E_y(x,t) + \frac{\omega_{pe}^2}{c^2}\nu_{e0}\int_0^t \exp\left[\nu_{e0}(s-t)\right]E_y(x,s)ds = 0, \quad (9)$$

where $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$. The integral-differential wave equation has been solved numerically by a code with the algorithm of average conceal difference method and composite Simpson integral method. Comparisons of the numerical results and Appleton formula show that the two results are matched well when $n_0/n_c < 1$, but are obviously different in $\nu_{e0}/f_0 < 1$ when $n_0 > n_c$. The n_0 is the maximum value of electron density profile, and $n_c = \pi m_e f_0^2/e^2$ is the critical electron density for wave frequency f_0. Because when the width fixed, the larger the ratio of n_0/n_c is, the bigger the gradient of the electron density (i.e. the inhomogeneity of the plasma layer)is. When the inhomogeneity is large enough, the WKB approximation is no longer correct. The detail of above and the dependencies of the passed wave behaviors (i.e. the phase shift, the reflectivity, the transmissivity and absorptivity) on the APP layer characteristics (width, electron density, and collision frequency) and microwave frequency are presented in [4].

3. two dimension case

When the microwave is oblique incident into a APP layer, the propagation of wave becomes a two dimension problem. In this case, the behaviors of microwave in APP will furthermore depend on the wave incident angle and its electric field polarization. So instead of Eq.(1), we have to accept the Maxwell equations to describe the behaviors of microwave in APP layer:

$$\frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} = -\frac{1}{\mu_0}\nabla \times \mathbf{E}(\mathbf{r},t), \quad (10)$$

$$\frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} = \frac{1}{\varepsilon_0}(\nabla \times \mathbf{H}(\mathbf{r},t) - \mathbf{J}(\mathbf{r},t)), \quad (11)$$

where μ_0 and ε_0 are the magnetic permeability and the electric permittivity of free space respectively. The disturbed plasma current $\mathbf{J}$ obeys electron motion equation

$$\mathbf{J}(\mathbf{r},t) = -en_e(\mathbf{r})\mathbf{u}(\mathbf{r},t), \quad (12)$$

$$m_e\frac{\partial \mathbf{u}(\mathbf{r},t)}{\partial t} = -e\mathbf{E}(\mathbf{r},t) - m_e\nu_{e0}\mathbf{u}(\mathbf{r},t). \quad (13)$$

These coupled set of equations have been solved numerically by the finite difference time domain (FDTD) method for Maxwell equations [8] and the finite difference method for the motion equation. The simulation is divided into S-polarization (TM) mode and P-polarization (TE) mode. In S-mode the electric field $\mathbf{E}$ is perpendicular to the incident plane, and the $\mathbf{E}$ is in the incident plane for P-mode. Then the dependencies of the passed wave behaviors (i.e. the phase shift, the reflectivity, the transmissivity and absorptivity) on the APP layer characteristics (width, electron density, and collision frequency) and microwave characteristics (frequency, incident angle and polarization) are presented. The detail of these numerical results have been shown in [5,6].

4. Summary

The main conclusions are:

1. When $n_0/n_c < 1$ (n_0 is the maximum value of electron density profile, and $n_c = \pi m_e f_0^2/e^2$ is the critical electron density), the Appleton formula works well, but when $n_0 > n_c$ we need take the numerical solutions instead of the Appleton formula.

2. The larger the microwave incident angle is, the bigger the absorptivity is. Because the path passed by wave increases with the incident angle.

3. The absorptivity of the P-polarization incident microwave is generally larger than the one of the S-polarization microwave.

4. The thicker the plasma layer is and the higher the average electron density is, the better the absorption is. So, we find that it is the product $\overline{n}d$ of the width d and average electron density $\overline{n}$ play key role in the absorption of microwave.

5. When $\nu_{e0} \simeq \omega_0 = 2\pi f_0$ the absorptivity reaches its maximum.

6. The less the average gradient of electron density is, the laragger the absorptivity is. Because large gradient of density causes large reflection of incident wave.

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