Dynamical Evolution of Rotating Stellar Systems: Fokker-Planck and N-body

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Abstract

We have carried out comparative study between direct N-body technique and Fokker-Planck approach for the dynamical evolution of rotating stellar systems. Because of limitations of computing resources, we have restricted to relatively small N ($\sim 10{,}000$) systems compared to realistic star clusters. We found that these two different approached generally give very similar results. Some minor differences are found to exist, but they can be understood because Fokker-Planck approach has to make some simplifying assumptions.

Key words: star clusters; N-body dynamics

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1. Introduction

There are two different approaches to study the dynamical evolution of stellar systems; statistical approach and direct integration of the N-body equations of motion. Among the statistical methods Fokker-Planck (henceforth FP) equation has been frequently used. The statistical methods make several simplifying approximations and have some limitations. More realistic simulations can be done by directly integrating the full equations of motions of all stars. However, the N-body integration requires huge amount of computing power. Thus FP approach is still very useful in understanding the dynamical evolution of stellar systems. The results of FP simulations are generally found to agree with N-body simulations for spherical and isotropic models.

An extension of FP model was done to include the effects of rotation by [3]. According to the Jeans theorem there should be three isolating integrals of motion for axisymmetric potentials [2]. However, we know the exact forms of only two of them: energy, E and angular momentum along the axis of rotation, J_z. Since the nature of the third integral of motion is not known clearly, the rotating FP model ignores it. Thus it is important justify such an ignorance through the comparison with realistic N-body calculations. Here we present our results on the comparison between N-body and Fokker-Planck approached for the dynamical evolution of rotating star clusters.

2. Fokker-Planck and N-body Models

2.1. *Numerical methods*

The rotating FP code was extended to go through the core collapse by including the binary heating effect by [4]. For direct N-body simulations of the dynamical evolution of rotating stellar systems, we have used the NBODY6 [1]. Although NBODY6 is capable of dealing with many more astrophysical ingredients, such as the existence of primordial bina-

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ries and stellar mass-loss due to stellar evolution, we have considered only the three-body binaries as heating source like rotating FP model.

2.2. *Initial models and boundary condition*

Initially rotating models called rotating King models are generated according to [5]. These models require two dimensionless parameters: the central potential parameter W_0 and rotation parameter ω_0 [see [4]]. We have used $W_0 = 6$ and chosen three different initial rotations ($\omega_0 = 0.0, 0.3$ and 0.6) to represent non-rotating, intermediately rotating and rapidly rotating models, respectively.

Because most globular clusters are bound to their host galaxy, they are tidally limited and stars escape from them through a tidal boundary. Among a few different implementations of modeling the tidal effects, we adopted the instantaneous removal of stars whose total energy becomes larger than tidal energy of the cluster. This approximation is known to be inconsistent with the realistic N-body treatment for small N models but the inconsistency becomes small for large N. We have modified the original NBODY6 code to mimic the tidal environment of the clusters modeled with the FP equation, which means that we remove stars immediately, whenever their energy exceeds the tidal energy as considered in FP models. In order to keep the density within the tidal radius as constant, the tidal radius decreases with time when there is mass-loss through the boundary. We have used the initial tidal radius obtained from FP models to compute the mean density which is kept constant.

The number of stars (N) in a cluster is one of the important parameters for the dynamical evolution of the cluster. We used $N = 10240$ for equal-mass models.

3. Results

In Fig. 1, we show the evolution of the central density and the central velocity dispersion of the cluster. The results obtained from FP model are displayed as thick solid lines. Although core collapse occurs at slightly different times, there are generally good agreements between FP and N-body results. N-body gives rather noisy data since there is a significant statistical fluctuations in physical parameters. In the early evolutionary stage the central density derived from N-body is less than the value obtained by FP for the model with $\omega_0 = 0.6$. This may be attributed to the ways of determining core radius: while FP determines radii along some specific zenith angle in order to reduce the flattening effect by rotation, N-body code assumes the system as a sphere and does not consider this flattening effect which is significant for initial phase of rapidly rotating models. Thus for the most rapidly model has largest discrepancy in the central density between N-body and FP.

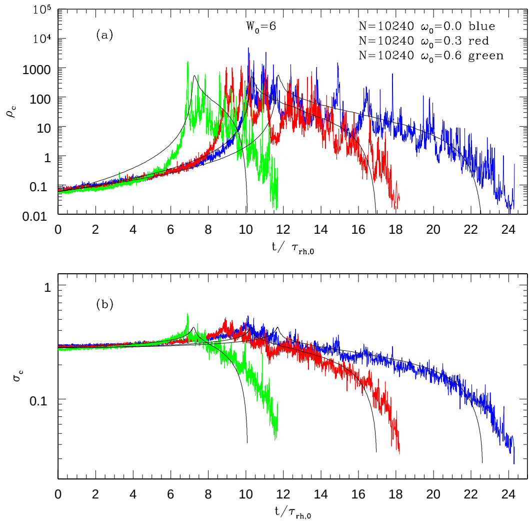

Fig. 1. Evolution of central density and central velocity dispersion for three different models with different initial rotation. FP models are shown with black solid lines while N-body models are shown as colored lines.

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