Instability of Pedestrian Flow in 2D Optimal Velocity Model with Attractive Interaction

Akihiro Nakayama [a,*], Katsuya Hasebe [b], Yūki Sugiyama [c]

[a] *Department of Physics and Earth sciences, University of the Ryukyus, Okinawa 903-0213, Japan*

[b] *Faculty of Business Administration, Aichi University, Miyoshi, Aichi 470-0296, Japan*

[c] *Department of Complex Systems Science, Nagoya University, Nagoya 464-8601, Japan*

Abstract

We incorporate an attractive interaction in two dimensional optimal velocity model and investigate the stability of homogeneous flow. There exists a new type of instability and a new phase appears. We also show the behavior of the flow in each phase by numerical simulations.

Key words: traffic; pedestrian; simulation; phase transition

1. Introduction

Traffic flow, pedestrian flow and some related systems present interesting phenomena and have been studied from the physical viewpoint. Recently we proposed a two dimensional optimal velocity (OV) model for pedestrian flow[1]. In this model, pedestrians are considered to be identical particles that have only repulsive interaction with other particles. By the linear analysis and numerical simulations, the stability condition of homogeneous flow is obtained and the phase structure is clarifed.

In this paper, we consider a model which has both repulsive and attractive interaction among particles generally. This modification does not change the form of the model. Therefore we can investigate the stability of homogeneous flow in the same way as Ref.[1], and can understand the property of the flow in a unified framework.

* Corresponding author.

Email address: nakayama@phys.u-ryukyu.ac.jp (Akihiro Nakayama).

The two-dimensional OV model is expressed by the equation of motion

$$\frac{d^2}{dt^2}\vec{x}_j(t) = a\left[\left\{\vec{V}_0 + \sum_k \vec{F}(\vec{r}_{kj}(t))\right\} - \frac{d}{dt}\vec{x}_j(t)\right], \quad (1)$$

$$\vec{F}(\vec{r}_{kj}) = f(r_{kj})(1+\cos\theta)\ \vec{n}_{kj}, \quad (2)$$

$$f(r_{kj}) = \alpha[\tanh\beta(r_{kj}-b)+c\ ], \quad (3)$$

where $\vec{x}_j = (x_j, y_j)$ is the position of the jth particle, and $\vec{r}_{kj} = \vec{x}_k - \vec{x}_j$, $r_{kj} = |\vec{r}_{kj}|$, $\cos\theta = (x_k - x_j)/r_{kj}$, $\vec{n}_{kj} = \vec{r}_{kj}/r_{kj}$. a is "sensitivity", which represents the strength of reaction of each particle. For simplicity, particles are supposed to move in the positive direction of the x-axis, and $\vec{V}_0 = (V_0, 0)$ is a constant vector which expresses the desired velocity. $\vec{F}$ expresses the interaction between particles.

The parameter c is varied in the range $-1 \le c \le 1$. $c = -1$ means that the interaction is repulsive , and $c = 1$ means that the interaction is attractive. In other value, the interaction is attractive at large distance and is repulsive at short distance. In numerical simulations and in the estimation of explicit values, we set $\alpha = 1/4$, $\beta = 2.5$, and $b = 1$.

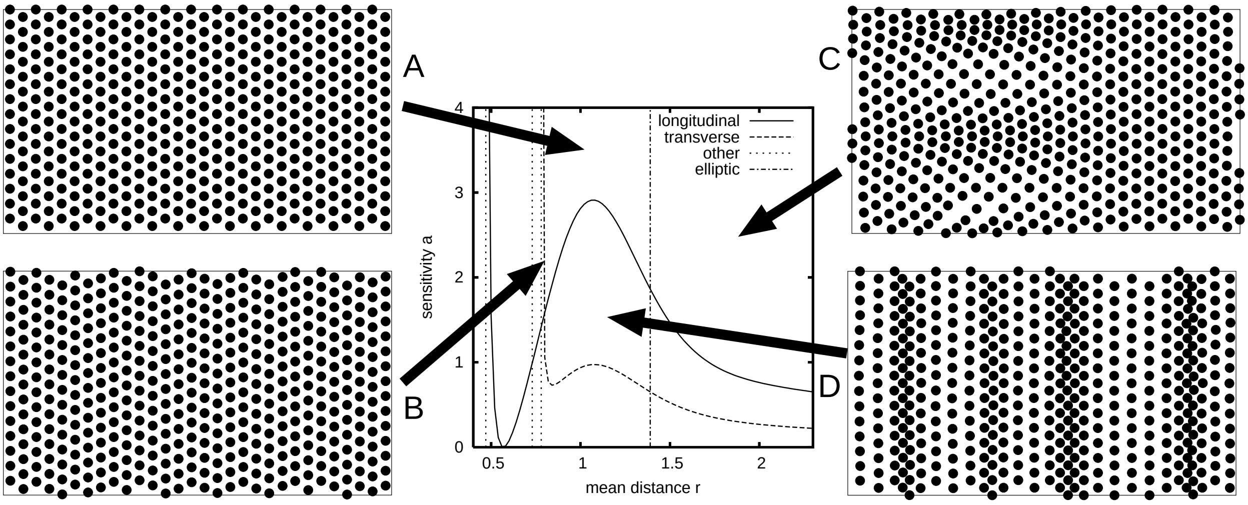

Fig. 1. Phase diagram for $c = 0$ and typical patterns of the flow. Solid and dashed lines represent the critical curves for the logitudinal and transverse modes along x-axis, respectively. Dotted lines ($r = 0.47,\ 0.73,\ 0.78$) are the critical lines for longitudinal and transverse modes in other directions. Dashed dotted line ($r = 1.39$) represents the critical line for elliptically polarized modes. Snapshots A, B, C and D represent breaking patterns of the flow in each phases.

2. Results

In the linear analysis, we consider three types of mode solutions, logitudinal modes, transverse modes and elliptically polarized modes, and find the stability conditions for these modes[2]. The properies of first two types of modes are the same as those in Ref.[1]. The instability due to the last mode solutions is the origin of a new phase, which appears only in the model with attractive interaction.

Figure 1 shows the results in the case of $c = 0$, where the interaction is attractive for large mean distance $r > 1$ and is repulsive for $r < 1$. In the region indicated by A, the homogeneous flow is stable. In the three regions, B (very narrow area), C and D, transverse modes along x-axis, elliptically polarized modes and longitudinal modes along x-axis are unstable, respectively. In numerical simulations, we can easily distinguish the behaviors in the three phases. The typical patterns are also shown in Fig.1. In other regions several modes become unstable simultaneously, and the borders among those phases can not be identified clearly by numerical simulations.

3. Summary

The attractive interaction considerably changes the phase structure. In the model which have repulsive interaction only, there is no unstable region due to the elliptically polarized mode. The instability of this mode narrows the stable region around the neutral position, that is, $f(r) = 0$, where no forces act on particles. In the case $c = 0$ the stability of elliptically polarized modes requires $0.73 < r < 1.39$. Pedestrians resemble particles binded by springs. However such an analogy is not always valid. In the case $c = 1$, the neutral position is $r = 0$ because of $f(0) = 0$, but the stability condition is $0.42 < r < 1.12$.

We find that the sparse homogeneous flow is not stable if the attractive interaction exists. Though the final state of the flow depends on the detail of the model, roughly speaking, particles make stable groups and there is almost no interaction among groups. The group formation of organisms may be explained by this mechanism.

Acknowledgements

This work is partly supported by a Grant-in-Aid for Scientific Research (C) (No.18540409) of the Japanese Ministry of Education, Science, Sports and Culture.

References

[1] A. Nakayama, K. Hasebe, Y. Sugiyama, Phy. Rev. E 71 (2005) 036121.

[2] A. Nakayama, K. Hasebe, Y. Sugiyama, to be published.