Dynamics of Interacting Two Particle Species on Scale-Free Networks

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Outline

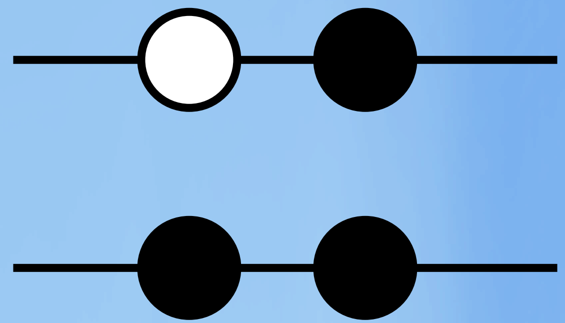
- Model for epidemic spreading
- Epidemic spreading on Scale-Free Networks
- Two species epidemic spreading on SF networks

References :

1 . "Asymetrically Coupled Directed Percolation Systems" by JDNoh and H. park, Phys. Rev. Lett. **94**, 145702 (2005).
2. "Epidemic Dynamics of Interacting Two Particle Species on Scale-Free Networks", by Y.-Y. Ahn, N. Masuda, H. Jeong, and JDNoh, cond-mat/0608461 (2006).

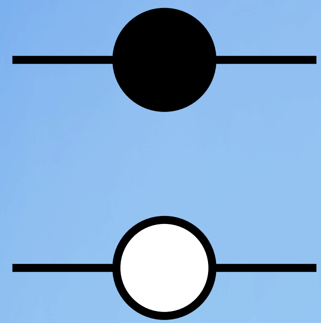
Contact Process or Susceptible-Infected-Susceptible model

- Epidemic dynamics



infection

(OX → XX)



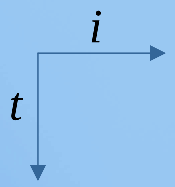
spontaneous healing

(X → O)

- Individual

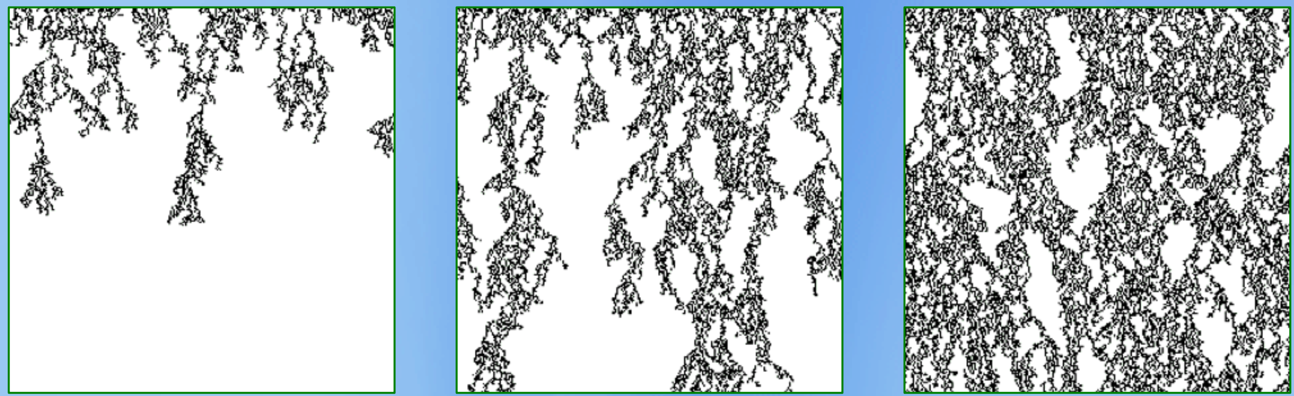
$$X \to O \to X \to \cdots \to O$$

Non-Eq. Phase Transition



X : ●

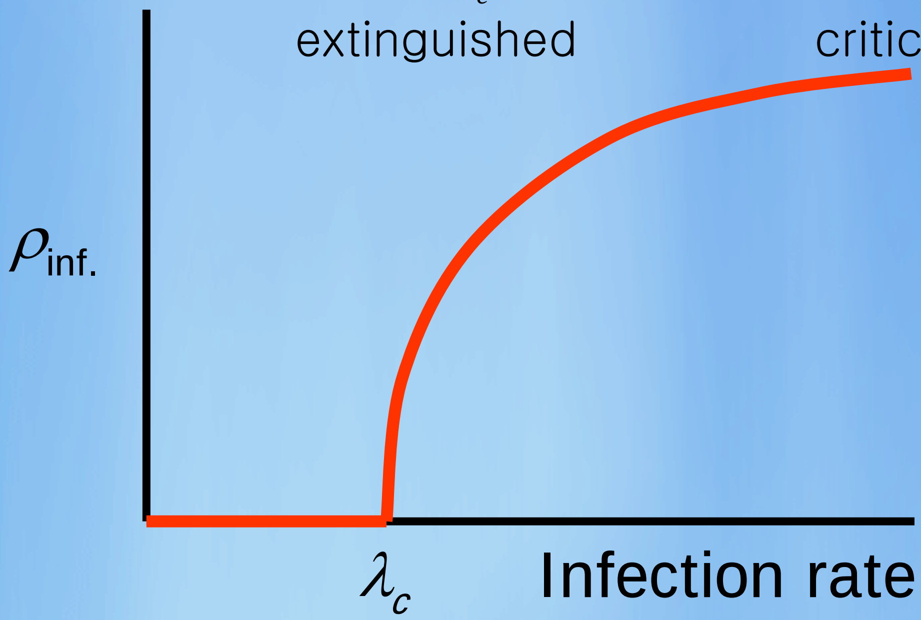
$\varnothing$: ○



$\lambda < \lambda_c$ extinguished

$\lambda = \lambda_c$ critical

$\lambda > \lambda_c$ everlasting



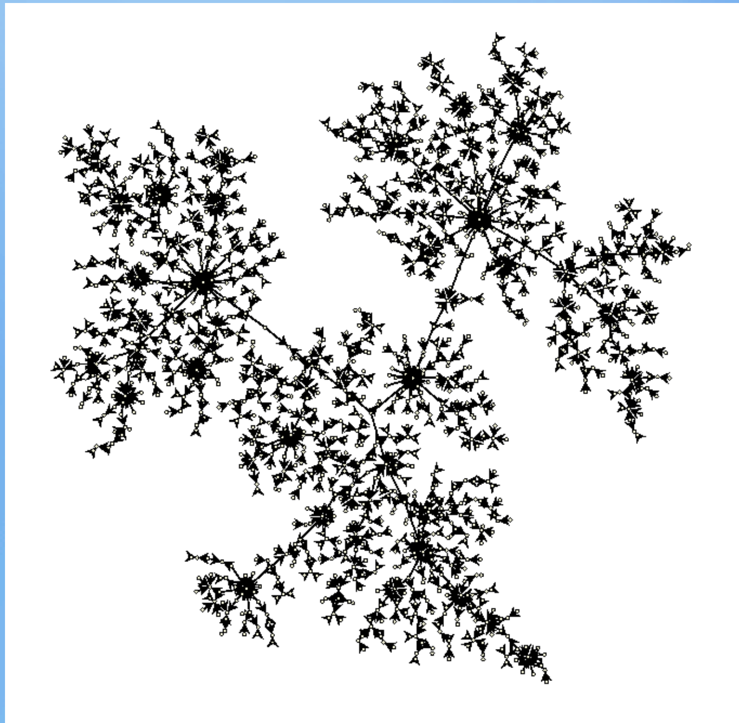
$$\rho_{\text{inf.}} \sim (\lambda - \lambda_c)^{\beta}$$

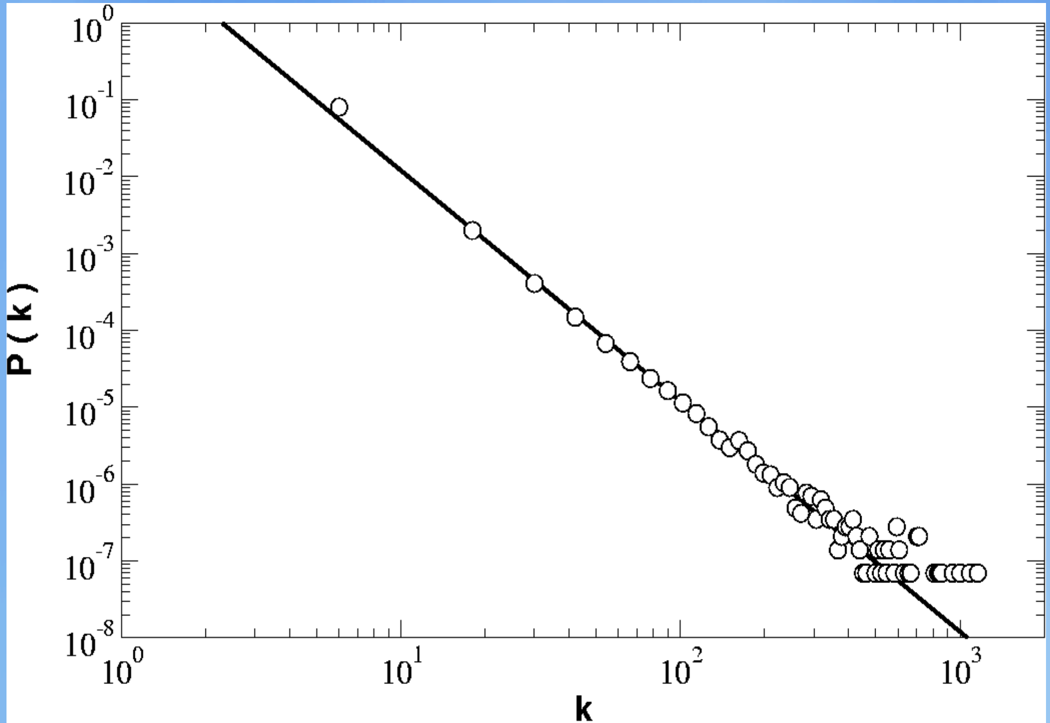
$$\xi_x \sim |\lambda - \lambda_c|^{-\nu_x}$$

$$\xi_t \sim |\lambda - \lambda_c|^{-\nu_t}$$

Directed **P**ercolation universality class

Scale-Free Networks





Power-law degree distribution $P_{deg.}(k) \sim k^{-\gamma}$

Ex) Internet, WWW,...

Epidemic on SF Net. (single species)

- Dynamics
  - Spontaneous healing with rate 1
  - Infecting all neighboring nodes with rate λ
- E.g., computer viruses on the Internet
- Effect of the heterogeneity of the underlying networks?

Mean Field Theory

- ρ_k : infected node density among nodes with degree k

- Rate equation

$$\dot{\rho}_k = -\rho_k + \lambda k (1 - \rho_k) \Theta$$

$$\text{with } \Theta = \int \frac{dk'}{\langle k \rangle} k' \rho_{k'} P_{\text{deg.}}(k')$$

- Stationary condition

$$\dot{\rho}_k = 0 \Rightarrow \rho_k = \frac{\lambda k \Theta}{1 + \lambda k \Theta}$$

- Self-consistency equation

$$\Theta = \int \frac{dk}{\langle k \rangle} P_{\text{deg.}}(k) \frac{\lambda k^2 \Theta}{1 + \lambda k \Theta}$$

Mean Field Results

- From the SC equation $$\Theta = \int \frac{dk}{\langle k \rangle} P_{\text{deg.}}(k) \frac{\lambda k^2 \Theta}{1 + \lambda k \Theta}$$
- Epidemic threshold $$\lambda_c = \langle k \rangle / \langle k^2 \rangle$$
- For SF networks with $$P_{\text{deg.}}(k) \sim k^{-\gamma}$$
  - $\gamma > 3$: finite $\langle k^2 \rangle$ ➔ finite threshold
  - $\gamma \leq 3$: infinite $\langle k^2 \rangle$ ➔ $\lambda_c = 0$

Immunization strategy?

Two species epidemics

- Epidemic spreading = competing process between virus (antigen) and vaccine (antibody)

- Competing asymmetric interaction
  - “A” induces “B”
  - “B” suppresses “A”

- Phase transitions and critical phenomena

[J.D. Noh and H. park, Phys. Rev. Lett. **94**, 145702 (2005).]

Two species epidemics on SF Networks.

- Dynamics
  - A particles
    - Spontaneous annihilation (A → 0)
    - Infection (OA → AA)
  - B particles
    - Spontaneous annihilation (B → 0)
    - spreading (OB → BB)
  - Interaction
    - Activation (A → AB)
    - Healing (AB → B)

Mean Field Theory

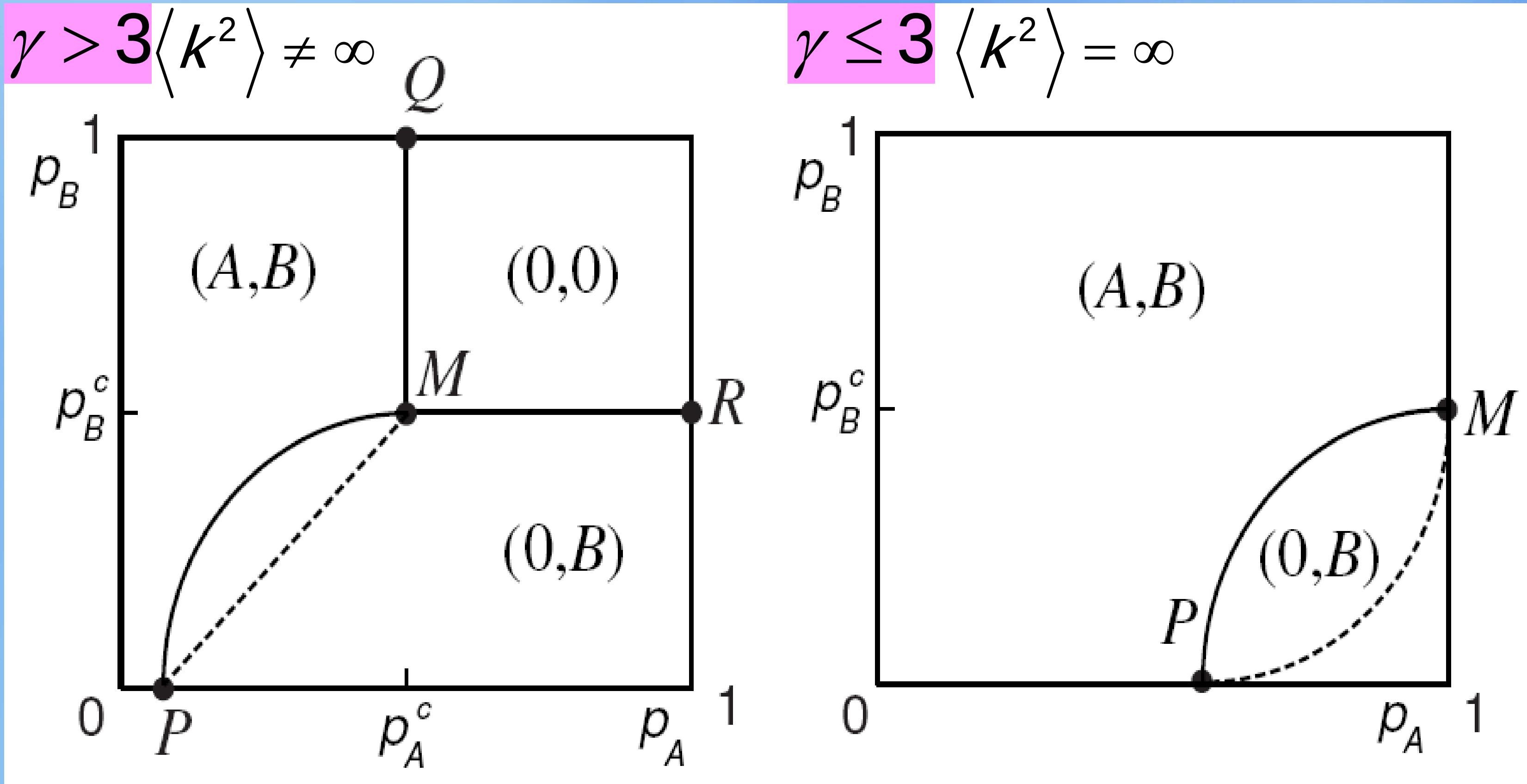
- a_k : A particle density on nodes with degree k

  b_k : B particle density on nodes with degree k

- Rate equation

$$\dot{a}_k = -p_A a_k + q_A(1-\lambda)k(1-a_k)\Theta_A(k) - \mu k a_k \Theta_B(k)$$
$$\dot{b}_k = -p_B b_k + q_B k(1-b_k)\Theta_B(k) + \lambda k(1-b_k)\Theta_A(k),$$

Phase Diagram



Summary

- Two species epidemic spreading model
- Critical behaviors
- Multi-species generalizations